1. The plane is parallel to $\overrightarrow{AB}$ and $\overrightarrow{AC}$. Note that we could have chosen any two other vectors. Let $O$ be the origin.

\[
\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} \\
= (3, 3, -2) - (2, -1, 5) \\
= (1, 4, -7) \\
\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} \\
= (-1, 6, 1) - (2, -1, 5) \\
= (-3, 7, -4)
\]

Hence, the vector form of the equation of the plane is $x = (2, -1, 5) + \lambda_1 (1, 4, -7) + \lambda_2 (-3, 7, -4)$ for all $\lambda_1, \lambda_2 \in \mathbb{R}$.

Note there are many other answers to this question, as long as you choose a position vector on the plane (either $A$, $B$ or $C$) and add it to the span of two non-parallel directional
vectors that the plane is parallel to.

2. You should draw a diagram to help you understand this question.

Let \( \overrightarrow{OA} \) be represented by \( a \), \( \overrightarrow{OB} \) be represented by \( b \), \( \overrightarrow{OC} \) be represented by \( c \) and \( \overrightarrow{OD} \) be represented by \( d \).

\[
\overrightarrow{OM} = \overrightarrow{OA} + \frac{1}{2} \overrightarrow{AB} \\
= \overrightarrow{OA} + \frac{1}{2}(\overrightarrow{OB} - \overrightarrow{OA}) \\
= \overrightarrow{OA} + \frac{1}{2}(b - a) \\
= \frac{b + a}{2}
\]

Similarly, \( \overrightarrow{ON} = \frac{c + d}{2} \).

\[
\therefore \overrightarrow{MN} = \overrightarrow{ON} - \overrightarrow{OM} \\
= \frac{c + d}{2} - \frac{b + a}{2} \\
= \frac{(c - b) + (d - a)}{2}
\]

Now notice that \( \overrightarrow{AD} = \overrightarrow{d} - \overrightarrow{a} \) and \( \overrightarrow{BC} = \overrightarrow{c} - \overrightarrow{b} \). But since \( \overrightarrow{AD} \parallel \overrightarrow{BC} \), we can let \( c - b = \mu (d - a) \) for some real \( \mu \).

\[
\therefore \overrightarrow{MN} = \mu \left( \frac{d - a}{2} \right) + \frac{d - a}{2} = \left( \frac{\mu + 1}{2} \right) (d - a)
\]

Hence \( \overrightarrow{MN} = \left( \frac{\mu + 1}{2} \right) \overrightarrow{AD} \), which is a scalar multiple of \( \overrightarrow{AD} \), so \( \overrightarrow{AD} \parallel \overrightarrow{MN} \).

Now consider the distance:

\[
|\overrightarrow{MN}| = \left| \left( \frac{\mu + 1}{2} \right) (d - a) \right| \\
= \left( \frac{\mu + 1}{2} \right) |d - a| \\
= \mu|d - a| + |d - a| \\
= \frac{\mu|d - a| + |d - a|}{2}
\]

which is the average of \( |\overrightarrow{AD}| = |d - a| \) and \( |\overrightarrow{BC}| = \mu|d - a| \).
3.

\[
\begin{pmatrix}
-1 & 2 & 1 & 1 & 0 \\
3 & -5 & -4 & -1 & 1 \\
1 & 3 & -6 & 9 & 5 \\
-1 & 3 & 0 & 6 & 7
\end{pmatrix}
\]

Perform the following row operations: \( R_2 = R_2 + 3R_1 \), \( R_3 = R_3 + R_1 \) and \( R_4 = R_4 - R_1 \).

\[
\begin{pmatrix}
-1 & 2 & 1 & 1 & 0 \\
0 & 1 & -1 & 2 & 1 \\
0 & 5 & -5 & 10 & 5 \\
0 & 1 & -1 & 5 & 7
\end{pmatrix}
\]

\( R_3 = R_3 - 5R_2, \) \( R_4 = R_4 - R_2 \)

\[
\begin{pmatrix}
-1 & 2 & 1 & 1 & 0 \\
0 & 1 & -1 & 2 & 1 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 3 & 6
\end{pmatrix}
\]

Let \( x_3 = \lambda \). We do this because column 3 is not a leading column. Now we start from the bottom and work upwards:

\[
3x_4 = 6 \\
x_4 = 2
\]

\[
x_2 - \lambda + 2x_4 = 1 \\
x_2 - \lambda + 4 = 1 \\
x_2 = -3 + \lambda
\]

\[
-x_1 + 2x_2 + \lambda + x_4 = 0 \\
-x_1 + 2(-3 + \lambda) + \lambda + 2 = 0 \\
x_1 = -4 + 3\lambda
\]
Hence:

\[
x = \begin{pmatrix} -4 + 3\lambda \\ -3 + \lambda \\ \lambda \\ 2 \end{pmatrix} = \begin{pmatrix} -4 \\ -3 \\ 0 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 1 \\ 1 \\ 0 \end{pmatrix}
\]

for \( \lambda \in \mathbb{R} \).
MATH1151 Algebra Test 1 2009 S1 v2B

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We cannot guarantee that our answers are correct - please notify us of any errors or typos at unswmathsoc@gmail.com, or on our Facebook page. There are sometimes multiple methods of solving the same question. Remember that in the real class test, you will be expected to explain your steps and working out.

1. Basically the same style as Test 1 2008 Version 2B.

\[ x = \begin{pmatrix} 5 \\ -2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 6 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -4 \\ -3 \end{pmatrix}, \quad \lambda, \mu \in \mathbb{R}. \]

Note that there are many forms for the answer to this question.

2. If we can express a vector through any two of the points (say for e.g. \( \overrightarrow{BP} \)) in terms of a vector through one of these points and the remaining point (so \( \overrightarrow{BM} \) or \( \overrightarrow{PM} \)), then \( B, P \) and \( M \) are collinear.

If we considered these vectors, then we would find \( \overrightarrow{BP} = \frac{2}{3} \overrightarrow{BM} \) implying \( B, P \) and \( M \) are collinear.

3. \[
\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = 1 \begin{pmatrix} 1 \\ -1 \\ 3 \\ -2 \end{pmatrix}.
\]